A GRAPHICOANALYTICAL METHOD OF ESTIMATING THE EFFICIENCY OF FINITE-PERMEABILITY SORBENT DESIGNS*

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UDC 621.53

In order to cope with problems concerning technically and economically optimized designs of suction devices with sorbent walls (cryosorption and cryogenic devices, various modifications of built-in discharge pumps, etc.), which are used more widely now in modern electrophysical apparatus construction, a graphicoanalytical method has been developed for estimating the performance of such devices at the user's plant. It is shown that the effective suction rate of a device with sorbent walls of an arbitrary profile and with a specific rate S_0 can be expressed as

$$S = \alpha S_{0\Sigma}, \tag{1}$$

where α is the reduction factor determined by the geometry of the sorption system and depending on the specific suction rate of the device as well as on the temperature and the molecular weight of the pumped gas, while $S_{0\Sigma}$ is the apparent suction rate of the sorption system, equal to the specific rate times of the sorbent surface area. Formulas are derived for the reduction factor of systems with a circular, a square, and a rectangular profile.

A table lists values of S_0 for cryogenic, cryosorptive, and magnetic-discharge suction systems operating on various different gases. The reduction factor is calculated and plotted as a function of the length-to-characteristic-dimension ratio for hydrogen, helium, methane, water vapor, and carbon dioxide at 4, 20, 77, 293, and 700°K. The specific rate has been selected as the parameter, its value ranging from 0.005 to 50 liters/cm^2 · sec, to cover the entire gamut of modern suction devices. The graphs show comprehensively the interrelation between the geometry and the vacuum characteristics on the one hand and the effective suction rate of a sorption system on the other.

Using the proposed method will simplify the design and the comparative evaluation of suction devices with sorbent walls.

A PARTICULAR MIXED PROBLEM IN CONVECTIVE MASS TRANSFER†

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A diffusion device is considered here where the converter element consists of two equally long coaxial cylinder-electrodes with opposite polarities. The cathode radius at the base r_1 is smaller than the

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respective anode radius r_2, i.e., $r_1 < r_2$. The density of the input current between the electrodes is assumed to be a function of z only. Assuming further that the operating requirements usually stipulated for chemotronic applications of such devices are met, the author defines the problem as that of determining the output current of the device when the latter operates in the mode of limiting diffusion current.

It can be shown that the conventional method of determining the output current from the known concentration of reduced electrolyte according to Fick's law will lead to considerable mathematical difficulties here. On the other hand, the problem here is definable as one of mixed boundary values for the differential equation of convective diffusion and it can, by means of a special kind integral transformation, be converted into another boundary-value problem admitting an exact solution for a large class of functions in both the boundary condition and the initial condition. The requirement is that these functions must be expandable into series with respect to the eigenfunctions $R_0(\lambda_k r)$ of the boundary-value problem

$$R'' + \frac{1}{r} R' + \lambda^2 R = 0, \quad R(r_1) = \left.\frac{dR}{dr}\right|_{r=r_2} = 0,$$

which is possible with almost all functions encountered in the theory of electrochemical converters.

The function which describes the sought output current must satisfy the equation

$$\frac{\partial U}{\partial t} = D\left(\frac{1}{r}\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2}\right) + \frac{D\mu}{r} + f(r, t)$$

and the additional conditions

$$\left.\frac{\partial U}{\partial r}\right|_{r=r_2} = U(r_1, t) = 0, \; t > 0,$$

$$U(r, 0) = \nu(r).$$

The formula for the output current becomes, in this notation,

$$i_{\text{out}}(t) = 2\pi r_1 kFD\Big\{\mu + \sum_{m=1}^{\infty}\lambda_m\Big\{\int_0^t\Big[a_m + b_m(\tau)\Big]\exp\Big[-\lambda_m^2 D(t-\tau)\Big]d\tau\Big\}R_0'(\lambda_m r_1) + \sum_{m=1}^{\infty}\lambda_m c_m \exp(-\lambda_m^2 Dt)R_0'(\lambda_m r_1)\Big\},$$

where F is the Faraday constant, D is the diffusivity, and

$$a_m = D\mu\left\| R_0(\lambda_m r)\sqrt{r}\right\|^{-2}\int_{r_1}^{r_2} R_0(\lambda_m \xi)d\xi,$$

$$b_m(t) = \left\| R_0(\lambda_m r)\sqrt{r}\right\|^{-2}\int_{r_1}^{r_2} f(\xi, t)R_0(\lambda_m \xi)\xi d\xi,$$

$$c_m = \left\| R_0(\lambda_m r)\sqrt{r}\right\|^{-2}\int_{r_1}^{r_2} \nu(\xi)R_0(\lambda_m \xi)\xi d\xi.$$

DETERMINING THE THERMOPHYSICAL PROPERTIES OF FLUIDS BY THE DIFFERENTIAL METHOD

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The laboratory apparatus and the test procedure have been described in [1]. In differential methods [1] the test specimen and the standard specimen are placed under the same conditions. This eliminates some measurement errors due to thermal contact resistances, partial heat losses, insufficient heater power, etc.

Lensovet Institute of Technology, Leningrad. Original article submitted July 16, 1971; abstract submitted March 20, 1972.

The theory behind this method is based on the equation

$$P = \frac{1+\alpha}{\theta} = \frac{1}{\operatorname{erfc} y - \alpha \operatorname{erfc} 3y + \alpha^2 \operatorname{erfc} 5y - \ldots} = f(\alpha,\ y), \tag{1}$$

where

$$\alpha = \frac{\varepsilon - 1}{\varepsilon + 1};\quad \varepsilon = \frac{\lambda}{b\sqrt{a}};\quad b = \frac{\lambda_B}{\sqrt{a_B}};\quad y = \frac{h}{2\sqrt{a\tau}}. \tag{2}$$

Applying expressions (2) to the standard specimen and to the test specimen of equal thicknesses h at the same instant of time τ, we obtain

$$a = a_A \left(\frac{y'_A}{y'}\right)^2;\quad \lambda = \lambda_A \frac{\varepsilon}{\varepsilon_A} \frac{y'_A}{y'};\quad \frac{y'_A}{y''_A} = \frac{y'}{y''}. \tag{3}$$

The value of ratio y'/y'' for fixed values of θ' and θ'' completely determines parameters ε and y'.

An analysis of Eq. (1) indicates that for $y \geq 0.7$ the function of two variables $P = f(\alpha, y)$ becomes a function of one variable $P = f(y)$, accurately within 0.5-1.0%, i.e., Eq. (1) becomes

$$P = \frac{1+\alpha}{\theta} = \frac{1}{\operatorname{erfc} y} = f(y). \tag{4}$$

In this way, the rigorous equation (1) can be tabulated in two stages:

1) $P = f(y)$ in the interval $0.7 \leq y < \infty$;

2) $P = f(\alpha, y)$ for the interval $0 < y < 0.7$.

Unlike the method shown earlier in [1], this method yields the thermophysical properties for the case $\alpha_A \neq 0$ without any simplifying assumptions. With an appropriate choice of material for the thermoreceiver B for the standard specimen A, one can determine the thermophysical properties of solid, liquid, and loose substances according to the single formula (4). The values of $\operatorname{erfc} y = 1 - \operatorname{erf} y$ can be picked from tables of the probability integral.

Using a known value of α_A, one finds $P_A = -(1 + \alpha_A)\theta_A$ for various values of θ_A and, subsequently, also the values of y_A necessary for the calculation of the thermophysical properties. If the numerical value of P_A does not appear in the first table, then it must appear in the second table $P = f(\alpha, y)$ and it will determine the value of y_A corresponding to the given values of α_A and θ_A.

After that, the test data are further processed according to the procedure given in [1].

$P = f(y)$ and $P = f(\alpha, y)$ tables for practical use are included in the article, also the thermophysical properties obtained by this method for polyethers and aqueous syrup solutions.

NOTATION

λ, λ_B are the thermal conductivities of the test material and of the thermoreceiver, respectively;
α, α_B are the thermal diffusivities of the test material and of the thermoreceiver, respectively;
b is the thermal activity of the thermoreceiver.

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HEAT AND MASS TRANSFER DURING AIR-AND-EVAPORATION COOLING OF A CYLINDER IN AN ANNULAR CHANNEL

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One possible method of air-and-evaporation cooling a cylinder in an annular channel [1] is considered, with simultaneous axial injection of air and wetting the lateral surface by a liquid.

A hydrodynamic analysis of this process shows that, while the liquid film here flows in the laminar mode, the two-phase mainstream can either be vortex-free or contain Taylor vortices. A chart of mainstream flow modes indicates the various characteristic regions.

With the aid of these results, an analysis of heat and mass transfer within each region is based on a structural procedure of determining the total thermal flux density

$$q_E = q_{conv} + q_{evap} + q_{sep},$$

where the components of thermal flux density are q_{conv} (convection), q_{evap} (evaporation), and q_{sep} (separation).

It is assumed here that in this case q_{conv} and q_{evap} can be calculated on the basis of the analogy between heat and mass transfer, while q_{sep} will be regarded as the remainder term in Eq. (1).

Thus, the problem has been reduced to setting up an algorithm for the calculation of q_{conv} and q_{evap}, based on known similarity laws of heat transfer during the flow of a homogeneous fluid in geometrically similar channels.

The basic layout of the test apparatus is described, including the rotating cylinder inside made of glass-fiber tubing with a two-layer printed-circuit heater coil mounted rigidly around its outside surface.

The test results are shown in the form of $q_E = f(t_w)$ relations, with t_w denoting the mean wall temperature of the rotating cylinder.

An analysis of test results covers two ranges. Within one range the measured and the calculated values of q_E agree within ±10%. Within the other range, at higher wall temperatures, the calculated values differ from the measured ones, namely: $q_{E,calc} > q_{E,test}$.

The discrepancy between $q_{E,test}$ and $q_{E,calc}$ can, according to this study, be explained by the existence of dry regions on the active rotating cylinder surface. A correction of calculated values by a coverage factor will ensure an agreement, within ±20%, between calculated and measured values.

The tests were performed within the following ranges of parameters: $10^2 \le Ta \le 35 \cdot 10^3$, $10^2 \le Re'' \le 7.8 \cdot 10^3$. $0.024 \le b/r \le 0.24$, $g' \le 35 \cdot 10^{-3}$ kg/m^2·sec, and $q_E \le 75$ kW/m^2.

NOTATION

$Ta = w^2 r b^3/\nu''^2$	is the Taylor number;
$Re = 2w_0'' b/\nu''$	is the Reynolds number;
w_0''	is the referred velocity of the gaseous phase, m/sec;
b	is the width of the annular gap, m;
w	is the angular velocity, sec^{-1};
r	is the inside radius of the cylinder, m;
g'	is the specific flow rate of liquid, kg/m^2·sec.

Superscripts

' denotes gas;

" denotes liquid.

Institute of Electrical Engineering, Novosibirsk. Original article submitted May 6, 1971; abstract submitted March 13, 1972.

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DETERMINING THE THERMAL RESISTANCE DURING HEAT AND MASS TRANSFER BETWEEN WATER AND AIR

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UDC 621.175.3

Merkel's method of the enthalpy potential is used widely in the analysis of simultaneously occurring heat and mass transfer processes in a water–air system:

$$dQ_\Sigma = dQ_\alpha + dQ_\beta = Gdi_G = Wc_W dt_W = \beta_x (i_{GW} - i_G)\, dF. \tag{1}$$

with the thermal resistance of the water film

$$dQ_\Sigma = \beta_x^0 (i_{Gg} - i_G) dF = \alpha_W (t_W - t_g)\, dF. \tag{2}$$

taken into account.

On the basis of the two-films theory, an equation is derived here relating the total thermal resistance of the system with the component resistances of air film and water film:

$$1/F\beta_x = 1/F\beta_x^0 + m/F\alpha_W. \tag{3}$$

Equation (3) is valid for evaporation and condensation in the given system. It is noted that coefficients α_W and β_X^0 can be found graphico-analytically by the Mickley–Mizushina method of stepwise curve plotting [1–2].

NOTATION

G, W	are the flow rate of air and water, respectively;
Q_Σ, Q_α, Q_β	are the total heat transferred, sensible heat, and latent heat, respectively;
h_G, h_{GW}, h_{Gg}	are the enthalpy of air in the mainstream, of air at $t_G = t_W$ and $\varphi_G = 1$, and of air at $t_G = t_g$ and $\varphi_G = 1$, respectively;
t_G, t_W, t_g	are the temperature of air, of water, and at the interface, respectively;
α_W	is the coefficient of heat transfer in the liquid film;
β_X	is the overall mass transfer coefficient (under the enthalpy difference as the motive force in the process);
β_X^0	is the coefficient of mass transfer in the air film;
F	is the area of transfer surface.

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DETERMINATION OF CRITERIAL NUMBERS DESCRIBING THE EFFECT OF MASS TRANSFER ON THE HEAT TRANSFER RATE IN SUBMERSED-COMBUSTION APPARATUS

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UDC 536.25

When aqueous solutions of nonvolatile substances are evaporating in submersed-combustion apparatus, then the heat transfer from the flue gases to the liquid surface is accompanied by a transport of vapor toward the gaseous phase. It is suggested that in such cases the effect of mass transfer on the heat transfer be evaluated in terms of dimensionless groups $\Pi g = \Delta p/p$, $\varepsilon_G = p_G/p$, and Pr_D [1]. At the same time, this effect is also described by the Gukhman number [2].

An analysis of the differential equations and of the boundary conditions has shown that, in the general case of nonadiabatic evaporation of a liquid, the criterial equation of heat transfer should contain not only the Πg, ε_G, and Pr_D numbers, but also the number $K = r/c\Delta t$. In the special case of evaporating aqueous solutions, this number may be replaced by the temperature factor T_G/T_L. In order to establish the significance of these numbers, an experimental study was made on a laboratory model of a submersed-combustion apparatus. The test results are generalized by the equation:

$$\alpha F \sim \Theta^{0,3}\, \varepsilon_G^{-0,54}\, \Pi g^{0,17}.$$

With a constant submersion depth of the bubble chamber and a constant flow rate of bubbling gases, it is permissible to disregard changes in the surface area of the interphase boundary.

NOTATION

- α is the heat transfer coefficient;
- F is the surface area of interphase boundary;
- p is the total pressure of flue gases;
- p_G is the partial pressure of dry components;
- Δp is the difference between partial pressures of water vapor;
- T_L is the absolute temperature of liquid in the apparatus;
- T_G is the absolute pressure of flue gases;
- r is the heat of evaporation;
- c is the specific heat of flue gases.

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Kiev Polytechnic Institute, Kiev. Original article submitted July 20, 1971; abstract submitted March 7, 1972.

SOLUTION OF THE EQUATION OF HEAT TRANSFER IN THE ANNULAR SPACE OF A ROTATING MUFFLE FURNACE BY THE N. K. KULIKOV METHOD

E. I. Trebukhin

UDC 536.24:669.041

The process of heat transfer in the annular space of a muffle furnace is described by the equation

$$\frac{dy}{dx} = -ay^4 - by + c$$

with the initial conditions $x_0 = 0$ and $y_0 = 1$.

In most cases one must know the temperature of gases only at the exit from the aggregate, i.e., at a single point. Therefore, it is worthwhile to solve the equation by the N. K. Kulikov method. This method of solution, which is outstanding in its simplicity, yields highly accurate results already in the zeroth approximation.

Replacing x by its end value x_e, we obtain the following solution to the original equation in the zeroth approximation:

$$y(x_k) = 1 - \frac{a+b-c}{\lambda(x_k)}\left[e^{x_k \lambda(x_k)} - 1\right].$$

Parameter λ can be approximated by the mean value of function ∂F(x, y)/∂y within the region G. For this particular equation, $F(x, t) = -ay^4 - by$.

On the basis of physical considerations, the maximum value of function y is in all cases equal to unity and its minimum value may be taken as equal to 1/2. Thus, the mean value of λ for the entire interval can be represented as

$$\lambda = p = -b - 2.25\,a.$$

The solution in the zeroth approximation on the x-interval from 0 to 1 will then be

$$y(x) = 1 - \frac{a+b-c}{p}(e^{px} - 1).$$

This solution is extremely simple and can be recommended for rough calculations. The error in determining the temperature of flue gases does not exceed 6%.

Parameter λ, as a function of x_e, can be determined more precisely on the basis of the mean-value theorem for a differential equation. With the designations

$$A = 1 + \frac{a+b-c}{p};\quad B = -\frac{a+b-c}{p},$$

we obtain the final expression

$$\lambda(x_k) = -b - 4aA^3 - \frac{4a}{(px_k - 1)\exp px_k + 1}\left\{3A^2B\left[\frac{\exp 2px_k - 1}{2} - (\exp px_k - 1)\right] + \frac{3AB^2}{2}\left[\frac{\exp 3px_k - 1}{3} - (\exp px_k - 1)\right] + \frac{B^3}{3}\left[\frac{\exp 4px_k - 1}{4} - (\exp px_k - 1)\right]\right\}.$$

In this particular case the error at the center of the x-interval from 0 to 1 does not exceed 1%, and at $x_e = 1$ the temperature of flue gases is determined almost exactly.

NOTATION

x is the dimensionless length of the muffle furnace;
y is the dimensionless temperature of flue gases;
a, b, c are the dimensionless parameters, constant for a given operating mode and determined on the basis of the thermal design of the aggregate.

Novosibirsk Polytechnic Institute, Novosibirsk. Original article submitted June 10, 1971; abstract submitted March 16, 1972.

HEAT TRANSFER BETWEEN MOVING BODIES IN CONTACT DURING METAL MACHINING

Yu. P. Rasputin

UDC 536.4

The problem is to determine the temperature field of the deformation zone OAB in cut metal (Fig. 1), across which heat transfer between a moving chip and the moving workpiece occurs during a machining operation. The author has calculated the thermal flux coming to the workpiece from this zone.

It is assumed in the formulation of the problem that the chip and the workpiece comprise two moving half-spaces which abut across the contact triangle OAB, that the process is steady, that the cut metal is a homogeneous and isotropic material, that its thermophysical properties are independent of the temperature, that the buildup deformation of cut metal occurs within a narrow zone regarded as a flat triangular source of uniform intensity, that heat conduction parallel to the plane of this source is negligible, and that the nose angle of the cutting tool is 90°.

Under these assumptions, the temperature field of the contact zone OAB is described as

$$\Theta_K = \frac{1}{2} + 0.56\,\mathrm{Ki}\sqrt{\frac{\eta}{\mathrm{Pe}_\eta}}, \tag{1}$$

where $\Theta_K = (T-T_2)/(T_1-T_2)$, with the temperature in the zone of basic deformation T_1 and the temperature of the workpiece prior to entering the contact zone T_2, the Kirpichev number Ki, the Peclet number Pe_η, and the dimensionless space coordinate η.

The solution is valid for $\mathrm{Pe}_\eta \gtrsim 10$.

Calculations based on formula (1) show that, within the practical range of machining conditions, the temperature at individual points in the zone of buildup deformation may reach 700°C or higher and thus degrade the superficial layer of the workpiece with a resulting high wear rate at the built-up edge at the back of the tool. Such an appreciable tool wear the built-up back edge in a machining configuration similar to the one shown here is often encountered in practice.

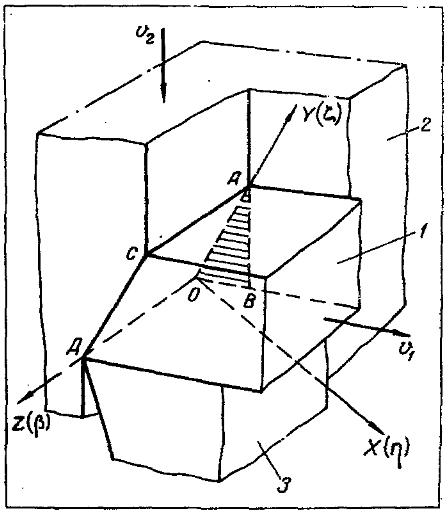

Fig. 1. Schematic diagram of a constrained machining operation: 1) chip; 2) workpiece; 3) tool.

The thermal flux transmitted from the zone of buildup deformation to the workpiece is, in dimensionless form,

$$\psi_2 = \frac{1}{2} + 0.75\,(1-\psi_1)\sqrt{\frac{\varepsilon}{\mathrm{Pe}_2\cos\gamma}},$$

where ψ_1 denotes the dimensionless flux transmitted to the workpiece from the zone of basic deformation in cut metal, ε denotes the per unit shear in the zone of basic deformation, γ denotes the limiting rake angle, and the Peclet number Pe_2 is a function of the cutter speed v_2.

By a comparison between measured and calculated values of thermal flux transmitted to the workpiece from the built-up cutting edge of the tool, this mathematical model is shown to reflect the fundamental laws governing the heat transfer from a chip to the workpiece within the zone of buildup deformation in cut metal.

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CONCERNING THE MECHANISM OF HEAT CONDUCTION AND THERMAL EXPANSION DURING HEATING

N. I. Nikitenko

UDC 53.01:536.1

According to the physical model of the heat conduction process analyzed in [1], the energy E (above the reference level) of particles in a body is proportional to the energy Q emitted by these particles per unit time:

$$Q=\varepsilon\dot{E}. \tag{1}$$

On the basis of relation (1), an integrodifferential equation of heat conduction is derived from which, as special cases, follow the Fourier equation and the hyperbolic equation describing high-rate heat transfer processes.

The energy flux of carriers emitted by a particle exerts a pressure on other particles in the body. This pressure is proportional to the energy flux Q and, therefore, to the energy of particles E. Simple mathematical operations yield a relation between thermal stresses and the temperature, as well as an expression for the thermal expansivity in agreement with Gruneisen's law.

Relation (1) yields the basic distribution laws of statistical physics. According to the concepts developed in quantum mechanics, particles of a body can emit energy in packets equal to or multiples of a quantum $h\nu$. We find the probability W_{mN} that a subsystem of N particles is at the energy level m and has the energy content E_{mN}. If L_N is the number of subsystems containing N particles each, then $L_{mN} = L_N W_{mN}$ such subsystems will be at the energy level m and will emit energy $Q_{mN} = \varepsilon L_N W_{mN} E_{mN}$ per unit time. If a subsystem, while emitting energy, jumps to the zero level (m = 0), then $B_{mN} = Q_{mN}/E_{mN} = \varepsilon L_N W_{mN}$ subsystems will leave level m as a result of energy emission.

All particles of the system emit the total energy E_c. It is distributed over the subsystems in proportion to their respective absorption cross sections σ_N. The subsystems of N particles at level m receive an energy $Q'_{mN} = \varepsilon L_N W_{mN} E_z \sigma_N/\sigma$, where $\sigma = \sum_N L_N \sigma_N$, and, consequently, $B'_{mN} = Q'_{mN}/h\nu$ subsystems will jump from level m to level m + 1. As a result of the same process, $B'_{m-1,N} = Q'_{m-1,N}/h\nu$ subsystems will jump from level m−1 to level m. In this way, the number of subsystems at level m ($\varphi_N = E_c\sigma_N/h\nu$) becomes

$$\frac{dL_N W_{mN}}{d\tau} = B'_{m-1,\,N} - B'_{mN} - B_{mN} = \varepsilon L_N \varphi_N \left[W_{m-1,\,N} - \left(1+\frac{1}{\varphi_N}\right) W_{mN} \right]. \tag{2}$$

Solving Eq. (2) for systems in equilibrium simultaneously with the equations of constant subsystems number and constant energy yields, after correspondence between statistical and thermodynamic quantities has been established, the canonical and the canonical in the large sense Gibbs distribution, followed by the Bose–Einstein and the Fermi–Dirac distributions. For a system in equilibrium we have

$$w_{mN} = \frac{1}{z_{mN}} \exp\left(-\frac{E_{mN}}{kt}\right),\ m=0,1,\ldots,\ M_N,\ M_N = \frac{E_c}{h\nu_N}, \tag{3}$$

where

$$z_{mN} = \alpha_{mN}(\varphi_N+1)\left[1+\frac{\varphi_N}{M_N-\varphi_N}\left(1+\frac{1}{\varphi_N}\right)^{-M_N}\right]; \qquad \alpha_{M_N N} = \frac{M_N-\varphi_N}{M_N(\varphi_N+1)};$$

$\alpha_{mN} = 1$ for m = 0, 1, ..., $M_N - 1$; and $\varphi_N h\nu_N = h\nu_N/[\exp(h\nu_N/kt)-1]$ is the mean energy of a subsystem containing N particles. When $M_N/\varphi_N \to \infty$, Eq. (3) yields the Maxwell–Boltzmann distribution.

The sign of Z in Eqs. (24) and (27) in the printed text is erroneous.

Institute of Engineering Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Original article submitted February 2, 1971; abstract submitted March 7, 1972.

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TRANSIENT HEAT CONDUCTION THROUGH A TWO-LAYER CYLINDER

M. Ya. Pekarskii, N. Yu. Taits, and Z. N. Golovina

UDC 536.21

In view of the complexity of the exact solution to the problem of heat conduction through a two-layer cylinder [1], its use for engineering calculations is very difficult. For this reason, an approximate solution to this problem is considered with linear boundary conditions and with a uniform initial temperature distribution. The solution is obtained by the method of averaging the functional corrections [2, 3]. The solution is given for the inertial and for the regular process modes. According to this method, for the inertial mode we let

$$\frac{\partial^2\theta_1}{\partial\rho^2} = \beta_1(\tau) \text{ and } \frac{\partial^2\theta_2}{\partial\rho^2} = \beta_2,$$

where

$$\beta_1(\tau) = \frac{1}{\rho_1 - \gamma(\tau)} \int\limits_{\gamma(\tau)}^{\rho_1} \left(\frac{\partial\theta_1}{\partial\tau} - \frac{1}{\rho} \cdot \frac{\partial\theta_1}{\partial\rho} \right) d\rho;$$

$$\beta_2 = \frac{1}{\mathrm{Ka}\,(\rho_2 - \rho_1)\,\tau_0} \int\limits_{\rho_1}^{\rho_2} \int\limits_0^{\tau_0} \frac{\partial\theta_2}{\partial\tau}\, d\rho d\tau.$$

with the relative temperature drop $\theta_i = (T_i - T_m)/(T_0 - T_m)$ $(i = 1, 2)$, the dimensionless time $\tau = a_1 h^2 t$, the dimensionless radius $\rho = hr$, $h = \alpha/\lambda_2$, $\mathrm{Ka} = a_2/a_1$, λ_i is the thermal conductivity, a_i is the thermal diffusivity, α is the coefficient of external heat transfer, t is the time, r is the radius, $\gamma(t)$ is the depth of thermal flux penetration, and τ_0 is the length of the inertial period. The temperatures in the layers are determined by the temperature θ_c in the contact zone:

$$\theta_1 = 1 - (1 - \theta_c)\left[1 - \frac{1}{2}\,\omega\,(\rho - \rho_1)\,\frac{\theta_c + \xi}{1 - \theta_c}\right]^2,$$

$$\theta_2 = \frac{\omega}{K_\lambda}\left[(1 + \rho_2 - \rho)\,\theta_c - \frac{(\rho - \rho_1)(1 - \rho_2 + \rho)}{1 - \rho_2 + \rho_1}\,\xi\right],$$

which is found from the equation

$$3\omega^2\tau = \frac{1}{2}\left[\left(\frac{1 + \xi}{\theta_c + \xi}\right)^2 - 1\right] - \ln\frac{1 + \xi}{\theta_c + \xi}.$$

Here

$$\omega = \frac{K_\lambda}{1 + \rho_2 - \rho_1}; \quad K_\lambda = \frac{\lambda_2}{\lambda_1};$$

$$\xi = (\rho_2 - \rho_1)\left[1 + \frac{1}{2}(\rho_2 - \rho_1)\right]\beta_2 = -\frac{(\rho_2 - \rho_1)\left[1 + \frac{1}{2}(\rho_2 - \rho_1)\right]^2}{\mathrm{Ka}\,(1 + \rho_2 - \rho_1)} \cdot \frac{1 - \theta_c(\tau_0)}{\tau_0}.$$

The length of the inertial period is determined from the expression

$$\tau_0 = \frac{1}{3\omega}\left[\frac{1}{2}\,\rho_1\left(1 + \frac{1}{4}\,\rho_1\omega\right) - \frac{1}{\omega}\ln\left(1 + \frac{1}{2}\,\rho_1\omega\right)\right].$$

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For the regular mode we let

$$\frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\theta_1}{\partial\rho}\right)=\beta_1(\tau),\quad \frac{\partial^2\theta_2}{\partial\rho^2}=\beta_2,$$

where

$$\beta_1(\tau)=\frac{1}{\rho_1}\int_0^{\rho_1}\frac{\partial\theta_1}{\partial\tau}d\rho;\quad \beta_2=\frac{1}{\mathrm{Ka}(\rho_2-\rho_1)(\tau^*-\tau_0)}\int_{\rho_1}^{\rho_2}\int_{\tau_0}^{\tau^*}\frac{\partial\theta_2}{\partial\tau}d\rho d\tau,$$

and then

$$\theta_1=\left(1+\frac{\rho_1^2-\rho^2}{2\omega\rho_1}\right)\theta_C+\frac{\omega(\rho_1^2-\rho^2)}{2\rho_1}\xi,$$

$$\theta_C+\xi=[\theta_K(\tau_0)+\xi(\tau_0)]\exp\left[-\frac{2\omega(\tau-\tau_0)}{\rho_1\left(1+\frac{1}{3}\omega\rho_1\right)}\right],$$

θ_2 and ξ are found from the respective expressions for the inertial mode.

A comparison with the exact solution shows that this approximate solution is sufficiently accurate for engineering purposes. As an example, the cooling of a cylindrical ingot under the skin layer is determined according to this method and the results indicate an appreciable effect of that layer on the cooling process.

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STEADY-STATE TEMPERATURE FIELD OF A RECTANGULAR PLATE WITH MIXED BOUNDARY CONDITIONS ON ONE SIDE

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UDC 536.2.01

In practical applications of thermophysics there arise problems of determining the temperature fields of bodies with mixed boundary conditions at the reference surface. The well-known analytical methods of solving such problems are difficult to use for specific cases, because they reduce to a solution of infinite systems of linear equations. The feasibility of solving such problems analytically without resorting to infinite systems is shown here on the example of heat transmission through a rectangular tetrahedron.

The temperature field of the tetrahedron is described by the Laplace differential equation:

$$\frac{\partial^2 t(x,y)}{\partial x^2}+\frac{\partial^2 t(x,y)}{\partial y^2}=0, \tag{1}$$

$$0\leqslant x\leqslant R,\quad -h_1\leqslant y\leqslant h_2$$

with mixed boundary conditions:

$$t(x,-h_1)=t_1, \tag{2}$$

$$t(x,h_2)=t_2, \tag{3}$$

All-Union Scientific-Research and Design-Engineering Institute of Complex Automation in the Oil and Gas Industry.

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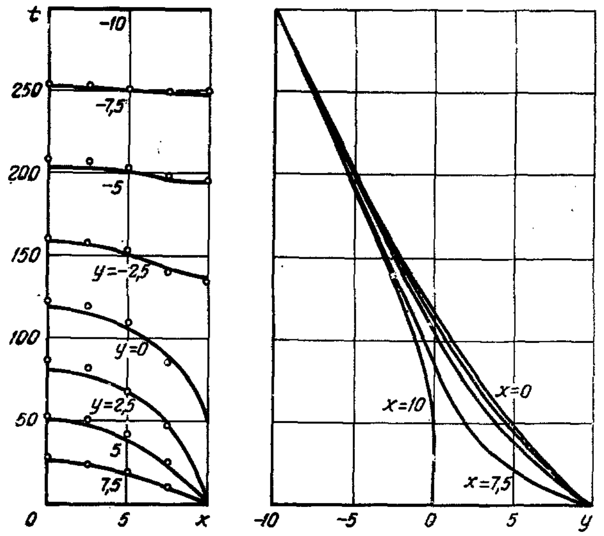

Fig. 1. Temperature field of a rectangular plate, determined numerically and analytically; the points represent values obtained by the Monte Carlo method, the solid lines represent values according to formulas (9) and (10). Coordinates x, y(cm), temperature t(°C).

$$\frac{\partial t(0,\ y)}{\partial x} = 0, \tag{4}$$

$$\frac{\partial t(R,\ y)}{\partial x} = 0, \quad -h_1 \leqslant y \leqslant 0, \tag{5}$$

$$t(R,\ y) = t_2, \quad 0 \leqslant y \leqslant h_2. \tag{6}$$

Equation (1) is solved by means of finite integral transformations for y > 0 and y < 0. The unknown integration constants are determined from the compatibility equations by eliminating one of the two kernels in the integral transformation:

$$c_1(0)\frac{h_1}{R} + t_1 + \frac{2}{R}\sum_{\sigma} c_1(\sigma)\cos\sigma x = t_2 + \frac{2}{R}\sum_{\nu} c_2(\nu)\cos\nu x, \tag{7}$$

$$c_1(0)\frac{1}{R} + \frac{2}{R}\sum_{\sigma} c_1(\sigma)\,\sigma\,\mathrm{cth}\,\sigma h_1\cos\sigma x = -\frac{2}{R}\sum_{\nu} c_2(\nu)\,\nu\,\mathrm{cth}\,\nu h_2\cos\nu x, \tag{8}$$

where $\sigma = \pi m/R$, $m = 0, 1, 2, \ldots$, $\nu = \pi(2n + 1)/2R$, and $n = 0, 1, 2, \ldots$

The final solution is

$$-h_1 \leqslant y \leqslant 0,$$

$$t(x,\ y) = t_1 - \frac{t_1 - t_2}{h_1 + \beta_1/\beta_2}\left[h_1 + y - \frac{4}{R^2}\sum_{\sigma}\sum_{\nu}(-1)^m\frac{1}{\nu^2 - \sigma^2}\,\frac{\mathrm{th}\,\nu h_1/\nu + \beta_1/\beta_2}{1 + \mathrm{th}\,\nu h_1\,\mathrm{cth}\,\nu h_2}\cdot\frac{\mathrm{sh}\,\sigma(h_1 + y)}{\mathrm{sh}\,\sigma h_1}\cos\sigma x\right]; \tag{9}$$

$$0 \leqslant y \leqslant h_2,$$

$$t(x,\ y) = t_2 + \frac{t_1 - t_2}{h_1 + \beta_1/\beta_2}\,\frac{2}{R}\sum_{\nu}\frac{(-1)_n}{\nu}\,\frac{\mathrm{th}\,\nu h_1/\nu + \beta_1/\beta_2}{1 + \mathrm{th}\,\nu h_1\,\mathrm{cth}\,\nu h_2}\cdot\frac{\mathrm{sh}\,\nu(h_2 - y)}{\mathrm{sh}\,\nu h_2}\cos\nu x; \tag{10}$$

$$\beta_1 = \frac{2}{R^2}\sum_{\nu}\frac{1}{\nu^3}\cdot\frac{\mathrm{th}\,\nu h_1}{1 + \mathrm{th}\,\nu h_1\,\mathrm{cth}\,\nu h_2}, \tag{11}$$

$$\beta_2 = 1 - \frac{2}{R^2}\sum_{\nu}\frac{1}{\nu^2}\cdot\frac{1}{1 + \mathrm{th}\,\nu h_1\,\mathrm{cth}\,\nu h_2}. \tag{12}$$

This solution is compared with the solution to the same problem by statistical tests (Monte Carlo method); the comparison is shown graphically. Both sets of values agree within the accuracy of the numerical method.

NOTATION

$t(x, y)$	is the temperature of the rectangular plate;
x, y	are the coordinates;
R	is the dimension of the plate on the x axis;
h_1, h_2	are the dimensions of the plate on the y axis;
t_1, t_2	are the temperatures on the boundaries of the plate;
σ, ν	are the significance of the nuclear transformation integral;
m, n	are the number of natural series.

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APPROXIMATE SOLUTION TO THE PROBLEM OF CONSOLIDATION UNDER CREEP IN A SWELLING SOIL BED

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UDC 624.131

The consolidation of a swelling soil is considered here in the V. A. Florin formulation of the problem, taking into account the rheological properties of the soil matrix. The consolidation equation, in conventional notation,

$$-a_0\gamma_i\frac{\partial^2 h(t)]}{\partial t^2}-\gamma_1\gamma_i(a_0+a_1)\frac{\partial h(t)}{\partial t}+\gamma[a_0+\beta'(1+\varepsilon_m)]\frac{\partial^2 H}{\partial t^2}$$
$$+\gamma\gamma_1[a_0+a_1+\beta'(1+\varepsilon_m)]\frac{\partial H}{\partial t}=k(1+\varepsilon_m)\left(\gamma_1\frac{\partial^2 H}{\partial z^2}+\frac{\partial^3 H}{\partial t\partial z^2}\right) \tag{1}$$

is solved for the initial conditions

$$H(z,\ 0)=0,\quad 0<z<h_0 \text{ and } -a_0\gamma_i\frac{\partial h}{\partial t}-a_1\gamma_1\gamma_i h+\gamma[a_0+\beta'(1+\varepsilon_m)]\frac{\partial H}{\partial t}+a_1\gamma_1\gamma H=k(1+\varepsilon_m)\frac{\partial^2 H}{\partial z^2} \tag{2}$$

and the boundary conditions

$$H=0\quad\text{for}\quad z=h(t),\quad \frac{\partial H}{\partial z}=0\quad\text{for}\quad z=0. \tag{3}$$

Introducing the variable $\xi = z/h(t)$ and applying the Fourier cosine transformation with finite limits, we obtain for the image function $\overline{H}_n$ when $h(t) = \alpha t + h_0$

$$(\alpha t+h_0)^3\frac{d^2\overline{H}_n}{dt^2}+[a(\alpha t+h_0)^3+c_n(\alpha t+h_0)]\frac{d\overline{H}_n}{dt}+c_n[\gamma_1(\alpha t+h_0)-2\alpha]\overline{H}_n+b_n(\alpha t+h_0)^3=0 \tag{4}$$

(a, b_n, and c_n depend on the calculated soil characteristics). The initial conditions for this equation are obtained from the initial conditions (2) after a Fourier cosine transformation.

A simplification of Eq. (4), with a_1 assumed much smaller than a_0 and 2α assumed negligibly smaller than $\gamma_1(\alpha t + h_0)$ at large values of t, yields the following approximate solution to Eq. (4):

$$\overline{H}_n=\int_0^t \exp\left[\gamma_1(z-t)\left\{A_n\exp\left[-\delta\frac{z}{z+m}(2n+1)^2\right]-b_n\int_0^z\exp\left[\delta\left(\frac{\tau}{\tau+m}-\frac{z}{z+m}\right)(2n+1)^2\right]d\tau\right\}dz, \tag{5}$$

$$n=0,\ 1,\ 2\ldots,$$

where $m = h_0/\alpha$, while A_n and δ are determined from the soil characteristics.

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The magnitude of the excess head is found by an inverse Fourier cosine transformation.

Formula (5) was used for calculating the threshold pressure in a soil bed with the following parameters: $a_0 = 5 \cdot 10^{-3}$ cm^2/kg, $a_1 = 2 \cdot 10^{-3}$ cm^2/kg, $\gamma_1 = 10^{-4}$ h^{-1}, $k = 10^{-7}$ cm/sec, $\beta' = 5 \cdot 10^{-3}$ cm^2/kg, $\varepsilon_m = 0.54$, $\gamma_i = 2$ tons/m^3, $\alpha = 0.275$ m/24 h, and $h_0 = 1$ m. The maximum excess head was found to occur at point z = 0, 254 m for a bed 300 m high. The approximate solution did not deviate from the exact solution by more than 5-6%.

INTERPOLATION METHOD OF DETERMINING THE THERMAL CONDUCTIVITY OF PHASES IN A TWO-PHASE HETEROGENEOUS SYSTEM

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UDC 536.2.083

In the proposed method, glass balls (the system matrix) with an unknown thermal conductivity are placed successively in two liquids with known thermal conductivities λ_1, λ_2 and the corresponding effective thermal conductivities λ_1', λ_2' are measured each time. When $\lambda_1 < \lambda$, then $\lambda_1' < \lambda$; when $\lambda_2 > \lambda$, then $\lambda_2' > \lambda$. By interpolation we find $\lambda' = \lambda$. The interpolation formula is

$$\lambda = \frac{B}{1 - A}, \tag{1}$$

where

$$A = \frac{\lambda_2' - \lambda_1'}{\lambda_2 - \lambda_1}, \quad B = \lambda_1' - A\lambda_1 = \lambda_2' - A\lambda_2.$$

Formula (1) applies when the relation $\lambda' = f(\lambda)$ is linear. Experiments described in the article show that this condition applies, as long as λ_1 and λ_2 do not differ too much from λ. In order to determine the thermal conductivity of a liquid, one pours that liquid (unknown thermal conductivity λ_1) into the vessel with balls. Having then measured the effective thermal conductivity of the system and knowing the values of A and B in Eq. (1) from the preliminary calibration test, one finds the thermal conductivity of the test liquid by the formula

$$\lambda_1 = \frac{\lambda_1' - B}{A}. \tag{2}$$

We note that in this case the thermal conductivity of the balls material does not have to be known.

This method is particularly convenient for determining the thermal conductivity of liquids. The main difficulty in that case is usually to eliminate the effects of convection and radiation. This makes it necessary to perform the measurements on thin layers and thus complicates the experiment. In the proposed method, filling the interstitial space between balls in a sufficiently deep vessel does not result in the formation of thick layers.

The interpolation method was used by the authors for determining the thermal conductivity of sugar solutions, spherical solid spheres, and granular material.

Lensovet Institute of Technology, Leningrad. Original article submitted July 19, 1971; abstract submitted February 24, 1972.

OVERRADIATION IN HIGH-TEMPERATURE HEAT EXCHANGERS

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UDC 536.27

The effect of overradiation along the channel of a counterflow heat exchanger on the temperature of flue gases and the chimney walls is evaluated here. The problem is formulated as follows: a heat exchanger consists of circular-section channels with an infinite thermal conductivity in the transverse directions and a zero thermal conductivity lengthwise. Through some channels flows a "hot" heat carrier, through other channels flows cold air in the opposite direction (recuperative heat exchanger).

For an analysis of a regenerative heat exchanger, we replace the latter by an equivalent recuperator.

Heat is transmitted from the hot gas to the wall and from there to the cold gas by convection (radiation from the gas is lumped into the heat transfer coefficient). The system of equations which describes the temperature field of the heat exchanger includes the equation of heat balance for "hot" and "cold" gases in a channel element and the equation of heat transfer at the wall, the latter equation taking into account radiation fluxes from all channel segments and from the space below as well as above the chimney. The boundary conditions in the problem stipulate the gas temperature at the channel entrances.

The equations were treated in the finite-difference approximation with gas temperature at the ends of the channel element assumed known and the wall temperature assumed equal to the mean.

Radiation terms in the equation were averaged over the channel element. The problem was solved by interpolation with respect to a radiation term and by the Newton method in the boundary-value stage. An appropriate program had been set up for using a "Minsk-22" digital computer.

In order to improve the convergence at large values of the radiation term, the authors applied the "damping" method with a following temperature approximation used only for establishing the sense and the magnitude of temperature changes and with the radiation term calculated accordingly from the corrected temperature field in terms of the respective preceding approximation.

Calculations were made for chimneys of blast-furnace air heaters. According to these calculations, overradiation results in a rise of both the air temperature and the smoke temperature. The apparent heat deficit is wiped out by a source associated with overradiation. Overradiation raises the temperature of both gases and of the chimney along the entire height of the heat exchanger and, if both gases have the same water equivalent, the temperature distribution along the chimney height becomes nonlinear with the knee of the curve toward higher temperatures.

An analysis of the results has shown that at smoke temperatures up to 1500°C in long channels (over 500 diameters long) the effect of radiation on the temperature field is small. As the blast temperature (temperature of the flue gases at the chimney entrance) rises, the effect of radiation becomes more appreciable and must be considered in the selection of chimney materials.

All-Union Scientific-Research Institute of Metallurgical Heat Engineering, Sverdlovsk. Original article submitted August 12, 1971; abstract submitted March 21, 1972.

ANALYSIS OF TEMPERATURE FIELDS AND THERMAL STRESSES IN THE REGULAR MODE

Yu. P. Kotel'nikov

UDC 536.12:539.319

It has been shown in [1, 2] that heating of bodies with internal heat sources will proceed in the regular mode, if the temperature is read relative to its steady-state level.

Such a method of regularization is adopted here to the case of a hollow cylinder whose both surfaces, inside and outside, participate in heat transfer under mixed boundary conditions of the third kind. By a modification of the integral method, a solution is obtained which involves the minimum number of dimensionless parameters and, at the same time, applies to transient heating of the cylinder from either inside or outside as well [3].

This solution makes it possible to extend well-known relations applicable to the regular heating mode also to thermal stresses and to represent those relations in an explicit analytical form convenient for practical use:

1. During the regular stage of heat transfer, the logarithm of the dimensionless temperature $\tilde{\theta}$ and the logarithm of dimensionless thermal stresses $\tilde{H}$ both change at the same rate at all points, this rate remaining constant in time:

$$\frac{\partial \ln \tilde{\theta}}{\partial \mathrm{Fo}} = \frac{\partial \ln \tilde{H}}{\partial \mathrm{Fo}} = -M,$$

where M denotes the dimensionless heat transfer rate

$$M = \frac{12\,[2\mathrm{Bi}_a + (2+\omega)\,\mathrm{Bi}_a\mathrm{Bi}_p + 2\,(1+\omega)\,\mathrm{Bi}_p]}{12\,(2+\omega) + (8+5\omega)\,\mathrm{Bi}_a + (2+\omega)\,\mathrm{Bi}_a\mathrm{Bi}_p + (8+3\omega)\,\mathrm{Bi}_p};$$

$$\tilde{\theta} = \frac{T_\infty - T}{\vartheta - T_0};\quad \tilde{H} = \frac{(1-\mu)}{E\alpha_T\,(T_0-\vartheta)}\,(\sigma_{t\infty} - \sigma_t);$$

and ω is an indicator of the wall curvature.

A relation between the rate of change of M and the Kondrat'ev number is derived for the given cylinder with a doubly-connected surface.

2. For any two points on the cylindrical wall (η_r, η_s) the ratio of dimensionless temperatures β and the ratio of dimensionless thermal stresses γ in the regular mode remain invariable:

$$\beta_{rs} = \frac{\tilde{\theta}_r}{\tilde{\theta}_s} = \frac{2 + \mathrm{Bi}_p + (2+\mathrm{Bi}_p)\,\mathrm{Bi}_a\eta_r - (\mathrm{Bi}_a + \mathrm{Bi}_a\mathrm{Bi}_p + \mathrm{Bi}_p)\,\eta_r^2}{2 + \mathrm{Bi}_p + (2+\mathrm{Bi}_p)\,\mathrm{Bi}_a\eta_s - (\mathrm{Bi}_a + \mathrm{Bi}_a\mathrm{Bi}_p + \mathrm{Bi}_p)\,\eta_s^2} = \mathrm{const}.$$

The ratio of thermal stresses on the surfaces of a hollow cylinder is

$$\gamma_{ap} = \frac{\tilde{H}_a}{\tilde{H}_p} = -\frac{(8+5\omega)\,\mathrm{Bi}_a + (2+\omega)\,\mathrm{Bi}_a\mathrm{Bi}_p - (4+3\omega)\,\mathrm{Bi}_p}{(4+\omega)\,\mathrm{Bi}_a - (2+\omega)\,\mathrm{Bi}_a\mathrm{Bi}_p - (8+3\omega)\,\mathrm{Bi}_p} = \mathrm{const}.$$

3. The time till the same dimensionless temperature and equal in magnitude thermal stresses are established at any two points of a hollow cylinder during the regular heating stage remains invariable:

$$\mathrm{Fo}\,(\eta_r;\ \tilde{\theta}) - \mathrm{Fo}\,(\eta_s;\ \tilde{\theta}) = \frac{\ln \beta_{rs}}{M};$$

$$\mathrm{Fo}\,(\eta_r;\ |\tilde{H}|) - \mathrm{Fo}\,(\eta_s;\ |\tilde{H}|) = \frac{\ln \gamma_{rs}}{M}.$$

The relations derived here simplify the solution of engineering problems and the analysis of thermal stresses.

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Original article submitted August 18, 1970; abstract submitted March 27, 1972.

EXPERIMENTAL DETERMINATION OF THE WET-BULB TEMPERATURE IN HIGH-TEMPERATURE GASES *

E. N. Bukharkin

UDC 536.248.2

During contact with a gas, water can be heated up to the wet-bulb temperature t_M at which the sensible heat from the gas to the liquid and the latent heat of evaporation from the liquid to the gas are in a dynamic equilibrium. It is usually assumed for water [1, 2] that t_M is equal to the temperature of adiabatic saturation t", i.e., the temperature of static equilibrium between gas and water after the processes of heat and mass transfer have been completed.

It has been established experimentally that t_M and t" are identical at moderate gas temperatures (50-300°C) [1, 2]. At high gas temperatures no such correspondence between t_M and t" could be confirmed experimentally. At the same time, with the gas parameters fixed, t_M depends on the trend of the h–s diagram (Appendix 1), which is determined by the ratio α/β, and is generally not equal to t" [3].

The problem of determining t_M is important in practical applications (e.g., heating a liquid by contact with a gas). Thus, t_M is determined here for the case of contact between water and a high-temperature gas. The design of the test apparatus and the test procedure have been described earlier in [4].

The experiment was performed in the following manner: the contact column was flooded with water at a temperature equal to t" of gases entering that column. The water temperature was measured along the height of the column, including its lower section. The appropriate gas parameters were regulated by an admixture of atmospheric air.

The experiment has shown that t_M and t" are almost equal (within 0.1°C) within the 240-1030°C temperature range and the 0.3-1.2 m/sec velocity range.

EQUILIBRIUM MOISTURE CONTENT IN ZINC TETRAOXYCHROMATE†

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UDC 66.047.1:667.622.1

The sorption equilibrium in zinc tetraoxychromate was studied at temperatures from 293 to 368°K and a relative humidity of air φ over the 0.1-0.9 range. Zinc tetraoxychromate $5ZnO \cdot CrO_3 \cdot 4H_2O$ is an inorganic pigment included in high-quality protective primer coatings. It is obtained by a synthesis of zinc oxide ZnO emulsion in water and chromic acid H_2CrO_4 solution at a 333°K temperature. For this study the authors used zinc tetraoxychromate paste from an industrial reactor vat with an initial moisture content 260% (referred to dry weight). The absolute dry weight of the material was determined after a measured specimen had been dried at a temperature T = 378 ± 2°K in the presence of P_2O_5, whereupon its chemical composition was established by volumetric analysis. The chemical content was found to be: 70.5 ± 0.5% ZnO, 17.8 ± 0.1% CrO_3, 11.7% $4H_2O$ (hydration moisture). The percent moisture content at equilibrium was measured in percent of dry weight.

The equilibrium function $f(u_p, \bar{\varphi}, T) = 0$ was explicated by an experimental determination of desorption isotherms for the given material. Sorption equilibrium was reached in an apparatus combining the principles of classical and dynamic test procedures, as shown in Fig. 1. The essential components of this

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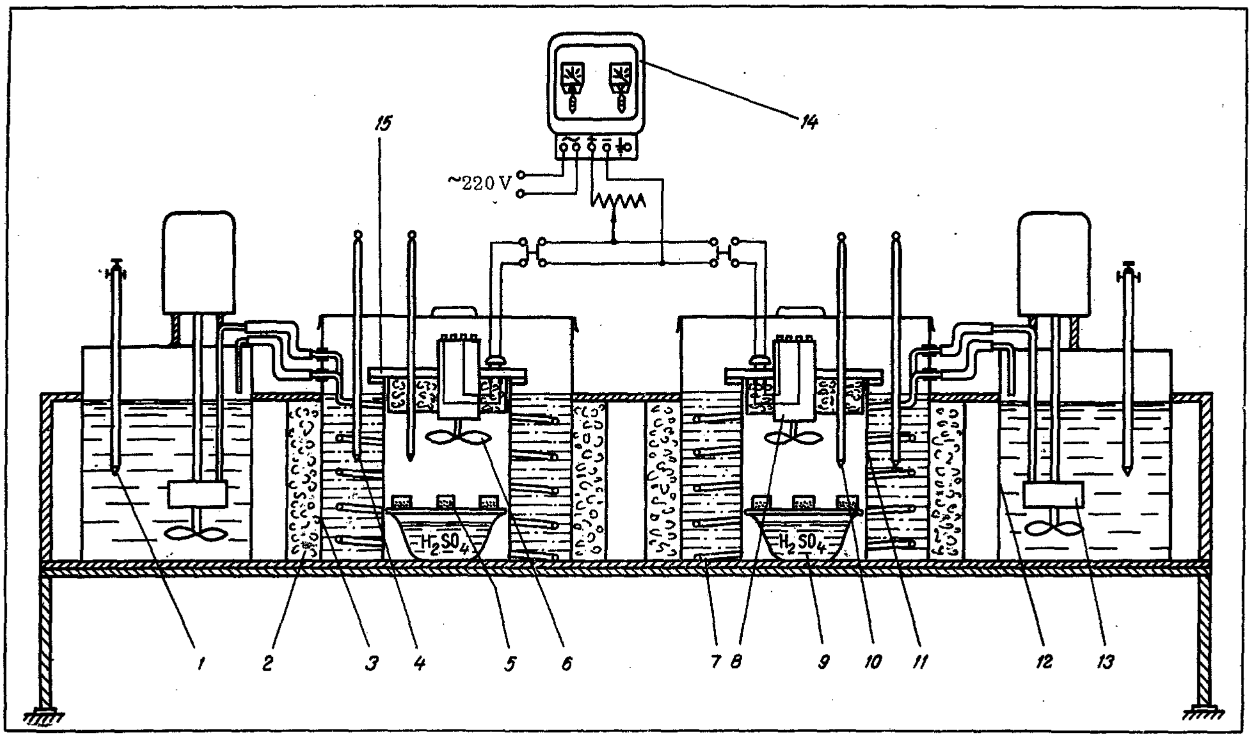

Fig. 1. Schematic diagram of the test apparatus for determining the desorption isotherms for zinc tetraoxychromate.

apparatus were: an exsiccator 11 covered hermetically with a lid 15, a dc electric motor 8, a rectifier 14, an impeller 6, an insulation box 2, a thermostat jar 3 containing a heater coil 7, a pump 13, a contact thermometer 1, thermometers 4 and 10, a vessel with H_2SO_4 solution 9, and pillboxes containing the test material.

The study covered also the effect of temperature on the stability of hydration moisture. It has been found that the hydration moisture is heat resistant up to T = 523°K and does not affect the equilibrium moisture content.

A family of desorption isotherms (T = 293, 308, 323, 348, 368°K) has been plotted for zinc tetraoxychromate on the basis of both test data and calculations. An analysis of these S-shaped curves indicates that this material is a capillary-porous colloid. Its bonded moisture amounts to approximately 3.2%. Moisture with different types of bond to the dry zinc tetraoxychromate matrix appears in following percentages: 42% hydration moisture, 31.5% monomolecular adsorption moixture, 20% purely polymolecular adsorption moisture, 25% partially polymolecular adsorption and partially capillary condensation moisture, and 18.8% moisture held by osmotic and wetting forces. This analysis indicates that moisture held by a strong bond, namely hydration and monomolecular adsorption moisture, constitutes over one-half (56%) of all bonded moisture.

The following empirical equation describes approximately the field of equilibrium moisture contents represented by the family of isotherms:

$$u_p = 21.8T^{-0.5}e^{0.8\varphi}, \tag{1}$$

where φ is expressed in fractions of unity. This formula, which contains three constants, yields, with a mean error of ±1.6%, the equilibrium moisture content in zinc tetraoxychromate over a wide range of variables covered in the tests: $0.1 \le \varphi \le 0.93$ and $293°K \le T \le 368°K$. The parameters in this empirical relation have been evaluated by the method of least squares.

STUDY OF GAS DYNAMICS IN VORTEX CHAMBERS

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UDC 533.697

Rotational motion of an incompressible viscous fluid in vortex chambers is analyzed under turbulent flow conditions. In order to describe the vortex hydrodynamics, the stream inside the chamber is tentatively divided into four zones: 1) jet flow near the lateral cylinder surface, 2) mainstream, 3) boundary layers at the end walls, and 4) region around the cylinder axis.

The energy loss in the jet, where a vortex is generated, is calculated by an approximate method on the basis of the law of momentum conservation applied to this zone. The equation of motion for the mainstream and the equation of the boundary layers at the end walls are solved simultaneously. On the basis of this solution, a method is then developed for calculating the velocity field and the pressure field in the mainstream and in the boundary layers at the end walls. The expressions for the tangential component of velocity and for the static pressure in the peripheral zone are:

$$\frac{v}{v_{in}} = \varepsilon\frac{R_0}{r} \text{ for } R_0 > r > r_1; \quad \frac{v}{v_{in}} = \varepsilon\frac{R_0}{r\left[1 + A\left(\frac{r_1}{R_0} - \frac{r}{R_0}\right)\right]} \text{ for } r_1 > r > r_0;$$

$$\frac{P}{0.5\rho v_{in}^2} = \frac{P_0}{0.5\rho v_{in}^2} + \varepsilon^2\left(1 - \frac{R_0^2}{r^2}\right) \text{ for } R_0 > r > r_1;$$

$$\frac{P}{0.5\rho v_{in}^2} = \frac{P_0}{0.5\rho v_{in}^2} + \varepsilon^2 - \frac{\varepsilon^2 R_0^2}{r_1^2} + \frac{2\varepsilon^2}{\left(1 + A\frac{r_1}{R_0}\right)^4}\left\{3A^2 \ln\frac{r}{r_1\left[1 + A\left(\frac{r_1}{R_0} - \frac{r}{R_0}\right)\right]}\right.$$

$$- A^3\left[\frac{r_1}{R_0} - \frac{r}{R_0\left[1 + A\left(\frac{r_1}{R_0} - \frac{r}{R_0}\right)\right]}\right] + \frac{1}{2}\left[\frac{R_0^2}{r_1^2}\right.$$

$$\left.+ R_0^2\frac{\left[1 + A\left(\frac{r_1}{R_0} - \frac{r}{R_0}\right)\right]^2}{r^2}\right] + 3A\left[\frac{R_0}{r_1} - R_0\frac{1 + A\left(\frac{r_1}{R_0} - \frac{r}{R_0}\right)}{r}\right]\left.\vphantom{\frac{1}{1}}\right\} \text{ for } r_1 > r > r_0,$$

where

$$A = \frac{2a_1c_1}{(a_1 - 1)}(2a_4)^{\frac{1}{4}}\left|\frac{2\pi R_0^2\varepsilon}{F_{in}}\right|^{\frac{5}{4}}\left(\frac{\nu}{\varepsilon v_{in}R_0}\right)^{\frac{1}{4}},$$

$$\varepsilon = \frac{v_1R_1}{v_{in}R_0} = \frac{1 - \frac{h}{2R_0}}{1 + 0.027\frac{(v'_{max})_{in}}{v_{in}}\mathrm{Re}_{in}^{-\frac{1}{4}}\frac{2\pi R_0 l_0}{F_{in}}},$$

$$r_1 = 1 - 3.972\left(\frac{F_{in}}{2\pi R_0^2\varepsilon}\right)^{1.124}\left(\frac{\varepsilon v_{in}R_0}{\nu}\right)^{0.225}.$$

These answers were verified experimentally in coaxial vortex chambers, in vortex chambers with gas discharging through orifices in the end walls, and in vortex chambers with a single inlet slot and gas injection tangentially through orifices distributed around the surface of the outer cylinder. The tests included measurements of the velocity field and the pressure field in the mainstream, in the boundary layers at the end walls, and at the lateral cylinder surface. These measurements have shown that at the end walls in vortex chambers there appear radial secondary currents which appreciably affect the vortex hydrodynamics. Analytical solutions based on a two-dimensional flow model which accounts for the interaction between mainstream and boundary layers agree closely with test data and can be used for calculating the velocities and the pressures in vortex chambers.

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NOTATION

R_0 is the radius of the vortex chamber;
r_0 is the radius of the outlet orifice;
h is the height of the inlet slot;
l_0 is the length of the vortex chamber;
v_{in} is the mean-over-the-mass velocity at the inlet slot;
V_0 is the tangential component of the inlet velocity;
U_0 is the radial component of the inlet velocity;
ε is the velocity maintenance factor;
r is the radial coordinate;
r_1 is the radius at which the radial spill current through the boundary layers at the end walls becomes equal to the rate of gas flow through the chamber;
ρ is the density;
P is the static pressure;
ν is the kinematic viscosity;
F_{in} is the total cross section area of the inlet slot;
$(v_{max})_0$ is the maximum inlet velocity;
$a_1 = 4.93$;
$a_4 = 0.269$;
$c_1 = 0.02255$.